Sci.Int.(Lahore),28(3),2309-2311 ,2016 ISSN 1013-5316;CODEN: SINTE 8

ON THE CONSTRUCTION OF FUZZY MAGIC GRAPHS

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ABSTRACT: *In this paper, we discussed the idea of fuzzy magic graphs. A fuzzy graph $G = (\alpha,\ \beta)$ is knows as a fuzzy magic graph if there exist two bijective functions $\alpha: A \to [0,1]$ and $\beta: A \times A \to [0,1]$ such that $\beta(ab) < \alpha(a) + \alpha(b)$ and $\alpha(a) + \beta(ab) + \alpha(b) = m(G)$ for all $a, b \in V(G\ )$ where $m(G) \in [0,1]$ is a fuzzy magic constant. Additionally, we investigated that fuzzy paths, fuzzy stars and fuzzy cycles are fuzzy magic graphs. To illustrate the applicability of fuzzy magic graphs we gave an illustrative example.*

Keywords: fuzzy magic graphs, fuzzy paths, fuzzy stars, fuzzy cycles

INTRODUCTION

Uncertainty in real life can be managed by fuzzy sets application. Fuzzy set was firstly introduced by [1]. Then, various researches added productive concepts to develop fuzzy sets theory like [2] and [3]. In 1987 Bhattacharya has succeeded to develop the connectivity notions between fuzzy bridge and fuzzy cut nodes. These researchers discovered a number of fuzzy resembling of graph theoretic definitions such as paths, cycles etc. A fuzzy graph has ability to solve uncertain problems in a range of fields that's why fuzzy graph theory has been growing rapidly and consider it in numerous applications of various fields. Exponential growth in the theory has been showing its utilization within mathematical sciences as well as it brings into play important role in technology. A fuzzy graph contains many properties similar to crisp graph due to generalization of crisp graphs but it diverge at many places.

A crisp graph G is an order pair of vertex-set V and edge set E such that $E \subseteq V \times V$. In addition $v = |V|$ is said to order while and $e = |E|$ is called size of the graph G respectively. In a crisp graph, a bijective function $\rho: V \cup E \to N$ that produced a unique positive integer (to each vertex and/or edge) is called a labeling [4]. Introduced notion of magic graph that the labels vertices and edges are natural numbers from 1 to $|V| + |E|$ such that sum of the labels of vertices and the edge between them must be constant in entire graph [5]. Extended the concept of magic graph with added a property that vertices always get smaller labels than edges and named it super edge magic labeling. Numerous other authors have explored diverse types of different magic graphs [6-8].

In section 2, we discussed basic definitions while in section 3, some new fuzzy magic labeling notions has been developed and explore fuzzy magic labeling for well-known graphs e. g. path, star and cycle. In Section 4, discussed a real life problem for demonstration of fuzzy magic graphs in last section gave the conclusion. In this paper finite and connected graphs are considered with standard terminologies and definitions. Reader may study [9-15] etc. for their contribution in graph theory.

PRELIMINARIES

Different real-world problems may be illustrated by union of different dots all the way joined through lines. A mathematical notion of such illustrations turned into the concept of a graph theory. In such illustration, the dots may stand for cities, computers, pieces of lands or group of persons and lines may be a symbol of roads linking between different cities, cable connection for network between different computers, bridges connecting pieces of land or correlation between people. In graph theory nodes are knows as vertices while the lines can be named arcs or edges. After few years of Zadeh's milestone concept fuzzy sets theory, Fuzzy graph theory developed as generliazation of graph theory by [2].

Let $\varphi: X \to Y$ be a function from X to Y, then φ is known as a fuzzy relation if φ is a fuzzy set of $X \times Y$. An order pair $\Gamma = (\alpha, \beta)$ of two functions α and β is called a fuzzy graph if $\alpha: X \to [0,1]$ and $\beta: X \times X \to [0,1]$ with satisfied a condition $(x, y) \leq \alpha(x) \wedge \alpha(y),\ \forall\, x, y \in X$. A fuzzy graph is known as a fuzzy path FP if there exist a series of different nodes $x_1, x_2, \dots, x_n$ such that $\alpha(x_i, x_{i+1}) > 0$; $1 \leq i < n$ and n is known as path's length. The successive order pairs (x_i, x_{i+1}) are said to be edge of the path. A cycle is a special case of a path if $x_1 = x_n$ for $n \geq 3$. $\bigwedge_i^n \beta(x_i, x_{i+1})$ is called the strength of a path FP. $d(x) = \sum_{x \neq y} \alpha(x, y)$ is said to be the degree of a vertex x in a fuzzy graph $\Gamma(\alpha, \beta)$.

Definition 1. A graph $\Gamma = (\alpha,\ \beta)$ is called a fuzzy label graph if $\alpha : A \to (0,1]$ and $\beta : A \times A \to (0,1]$ is bijective functions with the membership degree of vertices and edges are different and $\beta(a, b) < \alpha(a) \wedge \alpha(b)\ \forall\, a, b\ \in\ A$[16].

Example 1: Figure 1 showed α and β are bijective function, such that there is no same membership value for vertices and edges with the condition $\beta(a, b) < \alpha(a)\ \wedge\ \alpha(b)\ \forall\, a, b$

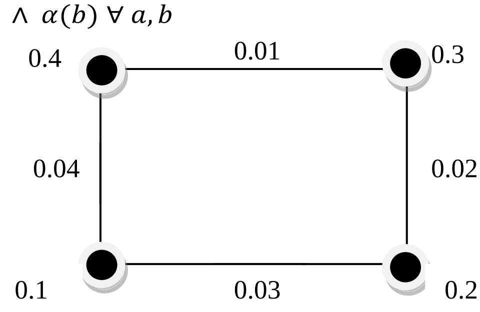

Figure 1. Fuzzy labeling graph

Definition 2. A fuzzy graph $G = (\alpha, \beta)$ is called a fuzzy magic graph if there are two bijective functions $\alpha: V \to [0\ 1]$ and $\beta: V \times V \to [0\ 1]$ with restricted the conditions $\beta(ab) < \alpha(a) + \alpha(b)$ and $\alpha(a) + \beta(ab) + \alpha(b) = \lambda(G) \leq 1$, where, $\lambda(G)$ is a real constant $\forall\, a, b \in G$

Definition 3. A fuzzy graph $\Delta:(\gamma,\delta)$ is said to be partial fuzzy sub graph of $G:(\alpha,\beta)$ if $\gamma(a) \le \alpha(b)$ and $\delta(a,b) \le \beta(a,b)$ for every a, b [17].

Definition 4. A fuzzy graph is called a fuzzy star graph, if there are two vertex sets X and Y with $|X| = 1$ and $|Y| > 1$, such that $\beta(y,x_i) > 0$ and $\beta(x_j,x_{j+1}) = 0$, $1 \le j \le n$ and denoted by $S_{1,n}$ [18].

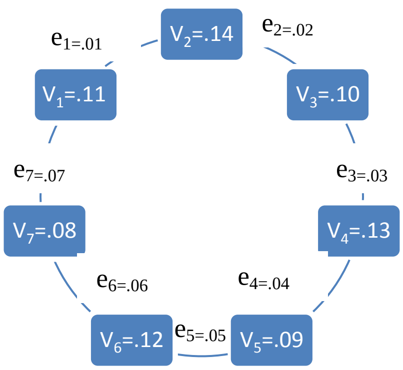

Figure 2: Fuzzy Magic Cycle for n= 7

SOME RESULTS ON FUZZY MAGIC LABELING GRAPHS

Theorem 1. For $n \ge 1$, a fuzzy Path P_n is consider as a fuzzy magic graph, where n is length of P_n

Proof: For fuzzy magic labeling of path, we see the following cases.

Case 1: When $n \equiv 1(mod2)$:

$\alpha(v_{2i-1}) = (2n - i)d$, where $1 \le i \le \frac{n+1}{2}$

$\alpha(v_{2i}) = \alpha(v_n) - id$, where $1 \le i \le \frac{n-1}{2}$

$\beta(v_{n-i}v_{n+1-i}) = \alpha(v_{n-1}) - id$, where $1 \le i \le n-1$ and,

$$
\begin{aligned}
d &= 10^{-1} && n = 1\\
&= 10^{-2} && 3 \le n \le 27\\
&= 10^{-3} && 29 \le n \le 285\\
&= 10^{-(j+4)} && 285 \times 10^{j} < n \le 285 \times 10^{j+1}\\
& && j \in N \cup \{0\}
\end{aligned}
$$

Thus, the fuzzy magic constant is

$$
\begin{aligned}
m(G) &= \alpha(v_{2i-1}) + \beta(v_{n-i}v_{n+1-i}) + \alpha(v_{2i})\\
&= (n-1)d + \alpha(v_{n-1}) + \alpha(v_n)
\end{aligned}
$$

Case 2: When $n \equiv 0(mod2)$:

$\alpha(v_{2i-1}) = (2n - i)d$, where $1 \le i \le \frac{n}{2}$

$\alpha(v_{2i}) = \alpha(v_{n-1}) - id$, where $1 \le i \le \frac{n}{2}$

$\beta(v_{n-i}v_{n+1-i}) = \alpha(v_n) - id$, where $1 \le i \le n-1$ and where,

$$
\begin{aligned}
d &= 10^{-1} && n = 2\\
d &= 10^{-2} && 4 \le n \le 28\\
d &= 10^{-3} && 30 \le n \le 284\\
d &= 10^{-(j+4)} && 284 \times 10^{j} < n \le 284 \times 10^{j+1}\\
& && j \in N \cup \{0\}
\end{aligned}
$$

Thus, the fuzzy magic constant is

$$
\begin{aligned}
m(G) &= \alpha(v_{2i-1}) + \beta(v_{n-i}v_{n+1-i}) + \alpha(v_{2i})\\
&= (n-1)d + \sigma(v_{n-1}) + \sigma(v_n)
\end{aligned}
$$

Theorem 2. For $n \ge 2$, a fuzzy star $S_{1,n}$ is a fuzzy magic graph.

Proof: Fuzzy magic labeling for $S_{1,n}$ is defined as follow:

$\alpha(v) = (n+1)d$ for $v \in V$

$\alpha(u_i) = \alpha(v) + id$ where $1 \le i \le n$

$\beta(vui) = \alpha(v) - id$ where $1 \le i \le n$

If $a = 3\sum_{j=0}^{l-1}[10^{j} - 1]$, $b = 3\sum_{j=0}^{l-1}[10^{j}]$ and l =total number of digits in n (i.e. if $1 \le n \le 9 \Longrightarrow l = 1$, $10 \le n \le 99 \Longrightarrow l = 2$, $100 \le n \le 999 \Longrightarrow l = 3$ and so on), then

$$
\begin{aligned}
d &= 10^{-1} && n = 2\\
&= 10^{-2} && 3 \le n \le 9\\
&= 10^{-l} && 10^{l-1} \le n \le a\\
&= 10^{-(l+1)} && b < n \le 10^{l} - 1
\end{aligned}
$$

Thus, the fuzzy magic constant is

$$
\begin{aligned}
m(G) &= \sigma(u) + \mu(vu_i) + \sigma(v)\\
&= (n+1)d + \sigma(v) - id + \sigma(v) + id\\
&= (n+1)d + (n+1)d + (n+1)d\\
&= 3(n+1)d
\end{aligned}
$$

Theorem 3. A fuzzy cycle Cn with $n \equiv 1$ mode 2, is a fuzzy magic graph.

Proof: Fuzzy magic labeling for Cn may be defined as follows:

$$
\beta(e) = \begin{cases} \beta(v_i v_{i+1}) = id, & 1 \le i \le n-1\\ \beta(v_1 v_n) = nd, & i = n \end{cases}
$$

$$
\alpha(v_{(n+2)-2i}) = \beta(v_1 v_n) + id, \quad 1 \le i \le \frac{n+1}{2}
$$

$$
\alpha(v_{(n+1)-2i}) = \alpha(v_1) + id, \quad 1 \le i \le \frac{n-1}{2}
$$

and

$$
\begin{aligned}
d &= 10^{-2} && 3 \le n \le 27\\
&= 10^{-3} && 29 \le n \le 287\\
&= 10^{-4} && 289 \le n \le 2849\\
&= 10^{-(j+4)} && 285 \times 10^{j} < n \le 285 \times 10^{j+1}\\
& && j \in N \cup \{0\}
\end{aligned}
$$

Thus, the fuzzy magic constant is

$$
m(G) = \alpha(v_1) + \beta(v_1 v_n) + \alpha(v_n)
$$

Remark: Degrees of any pair of vertices x, y in magic fuzzy graph always different from each other's and sum of degrees of nodes must be equal to twice the membership values of all arcs.

ILLUSTRATED EXAMPLE FOR FUZZY GRAPH

Let a company has five departments say D, D_1, D_2, D_3 and D_4. Department "D" coordinates with rest of all departments. Company gets a project and wants to complete it within a specific time period. Due to this company wishes to allocate work load according to individuals working abilities and size of departments with equalities. It is difficult to estimate individual working ability based on qualities, so as well for whole department. But in fuzzy conditions, any person or department working ability must always lie between [0 1]. Therefore fuzzy magic graph facilitate to allocate the work load between departments for achievement to complete the task within given time frame. We used star magic graph in this example, here boxes represent the departmental work load, edges between boxes shows the share work load between coordinator department and other departments. For completion of task, total work done by each department must be 100%.

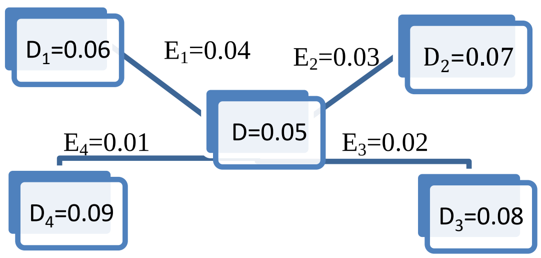

Figure 3: Fuzzy Magic star for n= 5

Table-1: % work distributed between departments

Departments	D	D1	D2	D3	D4
Dep.'s Work	33.33%	40%	46.67%	53.33%	60%
Sharing		E1	E2	E3	E4
Share work		26.67%	20%	13.33%	6.67%

CONCLUSION

Fuzzy graph theory may be used to solve the problems where uncertainties exist. Utilize it to model real systems in which involve uncertainty in different levels. The aims of fuzzy models are reducing the real time error which is not possible with traditional mathematical models, used in field of engineering and social sciences and hence utilize them in intelligent systems. In this paper, we defined the term of fuzzy magic graphs with fuzzy magic constant Moreover, some families of fuzzy graphs are proved as fuzzy magic graphs. However, in order to find more families of fuzzy magic graphs, the problem is open. We plan to explore our research work for new branch say soft fuzzy graph labeling and soft fuzzy magic labeling graphs, intuitionistic fuzzy label graphs and intuitionistic fuzzy magic graphs we also want to find application of these classes in different real life problems.

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